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Variation management of key control characteristics in multistage machining processes considering quality-cost equilibrium

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ABSTRACT

In multistage machining processes (MMPs), variations from key control characteristics (KCCs) continue to propagate and eventually accumulate to deviations in key product characteristics (KPCs). Therefore, the variation control of KCCs is significant to ensure the final product quality. In this paper, a variation management framework for KCCs in MMPs is established to address this issue. The new concept of variation management consists of process-oriented tolerancing and maintenance planning, and the optimal variation management strategy for each KCC is assigned based on its impact to the manufacturing system. The proposed framework deduces corresponding KCC variation distributions for previously unresearched locating schemes, thereby expanding the application scenarios of this method. For the quality specification constraints, geometric tolerances are integrated for the first time beside traditional dimensional tolerances, which expand the error scale of quality control. The modified Chebyshev goal programming (MCGP) approach is adopted to find the equilibrium point between quality and cost effectively. The superiority of the proposed method is verified by a case study of an automotive engine cylinder block MMP. The results show a remarkable improvement on the manufacturing system performance in terms of quality and cost.

1. Introduction

Multistage machining processes (MMPs) are widely used in manufacturing to obtain high-quality products by removing materials from the casting blank. Key product characteristics (KPCs) are critical features that describe a workpiece design so that it can satisfy particular functions. Therefore, meeting the quality specifications of KPCs in MMPs is a significant issue [1].

Tooling elements in MMPs are used to locate, clamp, or cut the unfinished workpieces. For a specific stage, machining features deviate from designed target values due to the imperfection of fixture locators or other tooling elements. If some of these deviating features are used as the datum features at downstream stages, their deviations will propagate and accumulate, ultimately affecting the machining precision of KPCs. Key control characteristics (KCCs), which contain process-related knowledge, are those tooling elements whose variations are the root cause of the KPCs' deviations [2]. Therefore, to maintain a high precision level of KPCs, variation management on KCCs should be necessary and helpful.

Describing the variation introduction and variation propagation from KCCs to KPCs is the basis of variation management. The stream of variation (SoV) theory is one of the most effective methods revealing the mapping relation between KCCs and KPCs [1]. Zhou et al. [3] explored the vector deviation representation and proposed a differential motion vector (DMV) based state space model to describe the SoV. This work derived detailed mathematical expressions of fixture variations and datum variations, providing a groundbreaking approach for modelling MMP variation propagation. Subsequently, this model was further expanded in fixture layouts [4–6], machining-induced variations [7,8], application objects [9–11] and geometric dimensioning and tolerancing (GD&T) integration [12]. By this time, the variation propagation rule from KCCs to KPCs in MMPs has been clear.

However, the KCC is inherently imperfect both statically and dynamically, which is a significant challenge for its variation management [13]. Static imperfection refers to the manufacturing deviation of KCC itself, while dynamic imperfection is the degradation of KCC due to wear or other factors in MMPs. Tolerancing and maintenance are two major tools to address static and dynamic imperfection. For MMPs,

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tolerancing determines the initial tolerance conditions for tooling elements, i.e. allocating allowable varying ranges. However, the allocated tolerances can only guarantee product quality in the early production if KCC degradation is not considered. Maintenance periodically renews the deteriorated KCCs to initial conditions to prevent out-of-tolerance workpieces due to dynamic imperfection. Therefore, the new concept of variation management comes into being, that is, the integration of tolerancing and maintenance.

Tolerancing, widely adopted in multistage assembly processes (MAPs), allocates final quality specifications to tolerances of each component, so as to minimize manufacturing cost on the basis of product functionality warranty [14–16]. The traditional product-oriented tolerancing focuses on assigning tolerance to product variables, i.e. assigning the assemblage's quality specifications to KPCs of each accessory. Most related researches build cost-tolerance functions with minimum cost as the objective, and solve them by optimization methods [17–19]. Although product-oriented tolerancing plays an important role in product optimization design, little attention is paid to process knowledge. Since variations in KCCs are the root cause of negative impacts on product quality, optimally assigning tolerances of KCCs throughout the manufacturing process has become a hotspot. Ding et al. [2] firstly established the process-oriented tolerancing approach for automotive body MAPs. However, this method fixed the replacement cycle of fixture elements for half a year, and did not fully utilize the dynamic degradation of KCCs. Chen et al. [13] expanded Ding's research, integrated process-oriented tolerancing with maintenance. The tolerances and maintenance strategies of assembly fixtures were comprehensively optimized, achieving the quality improvement and the cost reduction.

These two works laid the foundation for KCC variation management. However, they were concentrated on MAPs, where only assembly fixture's variation is of interest for KCC variation management purpose. In MMPs, there are more types of KCCs with different cost functions and different degradation models, such as cutting tool, various fixture elements in different fixture scheme, etc. Therefore, the KCC variation management in MMPs is relatively complicated and less explored. Huang and Shi [20] firstly developed a tolerance stackup model for MMPs, assigning tolerances to fixture locators by maximizing the variances of KCCs under the constrains of part quality specifications. Since the research on SoV was not mature enough at the time, this work adopted a two-dimensional variation propagation model. For all KCCs, only three point-to-surface fixture locators were considered, which were all assumed to be independent with each other and followed normal distributions. Based on the three-dimensional SoV model, Liu et al. [21] selected the optimal setup planning of MMPs by process-oriented tolerancing approach. Chen et al. [22] and Zhang et al. [23] optimized the KCC design through tolerance synthesis to improve quality and reduce cost. Since the dynamic degradation of KCC was not considered in above models, maintenance planning could not be integrated, all optimizations were achieved by changing the layout or process of manufacturing system. For the MMP from a mature manufacturing system, like automotive engine cylinder block machining, an optimal KCC variation management scheme, i.e. process-oriented tolerancing plus maintenance planning, is a better choice for most manufacturing companies [24–26]. Abellán-Nebot et al. [27] proposed a new process-oriented tolerancing approach. Two more KCCs, cutting tool and spindle temperature, were considered as new additions. The degradation models for locator and cutting tool were defined for maintenance integration. This method has achieved good performance in MMPs that only have point-to-surface locating scheme.

Although the simultaneous optimization for tolerancing and maintenance planning for KCCs in MMPs has achieved some successes, three main limitations can be identified in previous research works: (i) Applicable scenarios are limited. For the only applicable conventional 3-2-1 locating scheme, only the locators in the form of point-to-surface can be studied. Other fixture layouts, such as pin-hole locating scheme, although very common in MMPs, need to introduce new types of KCCs. (ii) Quality constraints only concern about dimensional tolerance, geometric tolerance is another important component of quality specification needs to be constrained. (iii) Finding the equilibrium between quality and cost is difficult in single-objective optimization with the quality as the constraint and the lowest cost as the goal. For the modified approach that defines the objective function as the sum of quality loss and variation management cost, the two components are incomparable, so there is no scientific method to determine the corresponding weights.

In this paper, a new variation management method is proposed for KCCs in MMPs with the consideration of quality-cost equilibrium. In response to the above limitations, the major contribution of this paper resides in three aspects: (i) The KCC variation management in MMPs is applicable to pin-hole locating scheme for the first time. Fixture pins are added as a new type of KCC in MMPs, whose tolerance-variation relation, degradation model and cost function are fully studied. (ii) For quality specifications of the workpiece, the geometric tolerance constrain is integrated beside traditional dimensional tolerance, which expands the scale of quality control. (iii) A bi-objective optimization model is proposed to find the equilibrium point between quality and cost, and a modified Chebyshev goal programming (MCGP) approach is adopted to solve it.

The outline of the paper is given as follows. In Section 2, the general framework and detailed procedures of the new method are presented. Section 3 validates the proposed method through an industrial case study of automotive engine cylinder block machining. Finally, the methodology is summarized in Section 4.

2. The proposed method

2.1. Overview and framework

Fig. 1 is the logic framework of the proposed method that demonstrates the interrelations between quality and cost in MMPs. The general architecture can be divided into three layers: the operation layer, the decision layer and the objective layer.

The operation layer describes the dynamic variation propagation from KCCs to KPCs. As introduced in Section 1, the variations in tooling elements are described by the process variables of KCCs. Through the variation propagation mechanism of MMPs, the variations of KCCs are transferred to KPCs, causing the final deviations and worsening the product quality. Suppose that the MMP has n tooling elements (including p locators/pins and q cutting tools) distributed in N stages, and there are m KPCs in the workpiece. The n KCC variations are described by $\mathbf{U} = [U_1 U_2 \cdots U_r \cdots U_n]^T$ and the deviations of m KPCs are denoted as $\mathbf{Y} = [Y_1 Y_2 \cdots Y_h \cdots Y_m]^T$. Based on the SoV theory, the KPC deviations $\mathbf{Y}$ can be represented as a function of $\mathbf{U}$. Section 2.2 will establish the relation and propose the variation propagation model in detail.

The decision layer is the output of the KCC variation management strategy. As mentioned in Section 1, the static and dynamic imperfections of KCCs are featured by tolerance and degradation respectively. $\mathbf{T} = [T_1 T_2 \cdots T_i \cdots T_p]^T$ denotes the allocated tolerance of locators/pins in KCCs, which determines the initial allowable varying ranges of $\mathbf{U}$. During the MMPs, $\mathbf{U}$ shifts dynamically with the continuous degradation of KCCs, which will worsen product quality until the failure that the KPC deviations exceed the quality specifications at a certain time. Maintenance can periodically restore the deteriorated KCCs to original states and avoid the failure. Since the preventive maintenance policy is adopted in this paper, $\mathbf{a}^f = \left[a_1^f a_2^f \cdots a_i^f \cdots a_p^f\right]^T$ is defined as the maintenance cycle of locators/pins and $\mathbf{a}^c = \left[a_1^c a_2^c \cdots a_j^c \cdots a_q^c\right]^T$ is defined as the replacement cycle of cutting tools. $\mathbf{T}$ and $\mathbf{a}$ are both key decision parameters that determine the state of KCCs in MMP, and their detailed

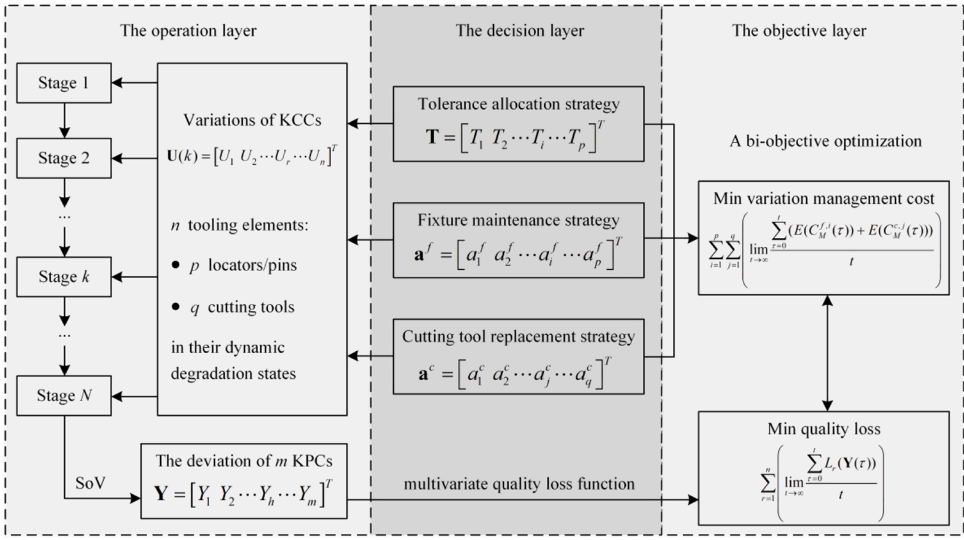


Fig. 1. The logic framework of the proposed method.

impact on the variations of KCCs will be demonstrated in Section 2.3.

The objective layer proposes the optimization goals for this paper, including minimizing variation management cost and minimizing quality loss. Generally, the tighter the tolerance or the higher the frequency of maintenance, the higher the cost of variation management, and the lower the quality loss. These two objectives conflict with each other. Therefore, a bi-objective optimization method is proposed to find the equilibrium point between quality and cost. The definition of objective functions, constraints, and optimization model will be illustrated in Section 2.4.

2.2. State space model of KCC variation propagation

2.2.1. Vector variation representation

The deviation of a feature can be represented by DMV of its own coordinate system (CS) w.r.t. another CS [3]. For the i^{th} feature of the workpiece, the DMV is defined by $\mathbf{x}_i^R = \left[(\mathbf{d}_i^R)^T \quad (\boldsymbol{\theta}_i^R)^T \right]^T$ w.r.t. reference CS, where $\mathbf{d}_i^R = \left[\Delta x_i^R \quad \Delta y_i^R \quad \Delta z_i^R \right]^T$ contains three small translation deviations and $\boldsymbol{\theta}_i^R = \left[\Delta \alpha_i^R \quad \Delta \beta_i^R \quad \Delta \gamma_i^R \right]^T$ contains three small rotation deviations.

For a MMP with N stages, assuming that there are W quality features involved in the variation propagation and the deviation of the i^{th} feature w.r.t. reference CS at stage k is a DMV $\mathbf{x}_{k,i}^R$ $(i = 1, 2, \ldots, W)$, the state vector $\mathbf{X}(k) = \left[\left(\mathbf{x}_{k,1}^R\right)^T \quad \left(\mathbf{x}_{k,2}^R\right)^T \quad \ldots \quad \left(\mathbf{x}_{k,W}^R\right)^T \right]^T$ is a stack of DMVs that represents the deviations of all quality features after stage k.

2.2.2. Variation contributors and propagation model

A clear understanding of variation propagation mechanism is the basis of KCC variation management. Three main variation contributors among MMPs are identified in this paper: fixture variations, cutting tool variations and datum variations. These variation contributors and their propagation can be abstracted as shown in Fig. 2 by state space representation.

Fig. 3 shows the two most representative locating schemes in MMPs, namely conventional 3-2-1 locating scheme and pin-hole locating scheme. Therefore, the fixture includes two types of KCC: locators and pins, which introduce variations into MMPs in different ways because of the difference in contact form, tolerance-variation relation and degradation model.

For the cutting tool, it gradually wears out during the process. Unlike the fixture, it is assumed that the cutting tool is perfect in its initial state, that is, the cutting tool variations only come from degradation. Therefore, maintenance planning is necessary for cutting tool but tolerancing is not needed.

Datum variations exist when the features produced by upstream stages are used as datum features. Although datum features do not introduce KCC directly, they play a vital role in the propagation of KCC variations.

The state space model is developed to describe the variation propagation in MMPs as a state equation and a measurement equation:

$$\mathbf{X}(k) = \mathbf{A}(k-1)\cdot\mathbf{X}(k-1) + \mathbf{B}(k)\cdot\mathbf{U}(k) + \mathbf{w}(k) \tag{1}$$

$$\mathbf{Y}(k) = \mathbf{C}(k)\cdot\mathbf{X}(k) + \mathbf{v}(k) \tag{2}$$

where $\mathbf{U}(k)$ is the random variable of introduced KCC variations at stage k, including fixture variations and cutting tool variations; $\mathbf{Y}(k)$ is the

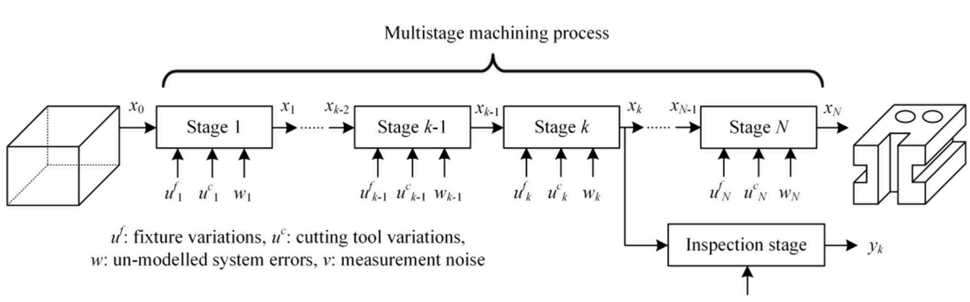
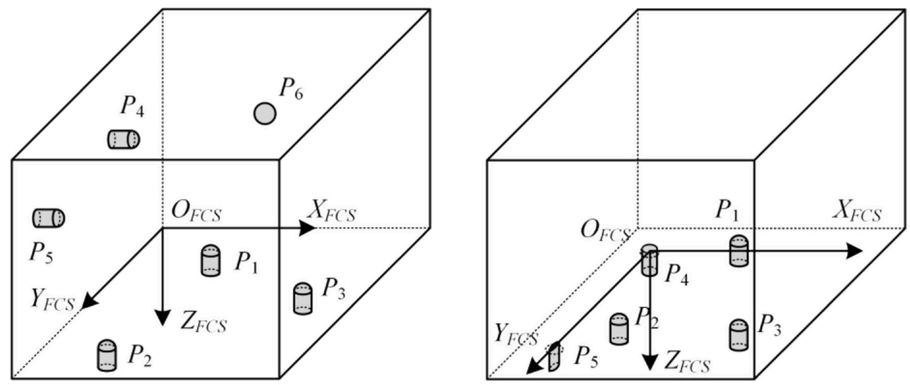


Fig. 2. Variation propagation by state space representation.



(a) conventional 3-2-1 locating scheme (b) pin-hole locating scheme

Fig. 3. The two most representative locating schemes in MMPs.

KPC deviation vector of stage k; $\mathbf{w}(k)$ and $\mathbf{v}(k)$ are the un-modelled system errors and the measurement noise respectively. The coefficient matrices $\mathbf{A}$, $\mathbf{B}$ are determined by process knowledge, such as the datum transformation between stages and the KCC layouts on individual stages, and $\mathbf{C}$ is determined by the position of KPCs and measurement datum features. The detailed expressions can be derived in [3].

Therefore, in the state equation, $\mathbf{A}(k-1)\cdot\mathbf{X}(k-1)$ represents the datum variations introduced from previous stages, and $\mathbf{B}(k)\cdot\mathbf{U}(k)$ represents the deviations introduced at current stage due to fixture and cutting tool. In the measurement equation, $\mathbf{C}(k)\cdot\mathbf{X}(k)$ represents the KPC deviations w.r.t. measurement datum.

Generally, product quality is evaluated by the KPC deviations at the final stage, that is, $k = N$ in the state space model. Substituting Eq. (1) into (2),

$$\mathbf{Y}(N) = \sum_{k=1}^{N} \mathbf{C}(N)\boldsymbol{\Phi}(N,k)\mathbf{B}(k)\mathbf{U}(k) + \mathbf{C}(N)\boldsymbol{\Phi}(N,0)\mathbf{X}(0) + \varepsilon \tag{3}$$

where the state transition matrix $\boldsymbol{\Phi}(N,k) = \mathbf{A}(N-1)\mathbf{A}(N-2)\cdots\mathbf{A}(k)$ and $\boldsymbol{\Phi}(k,k) = \mathbf{I}$.

Setting the initial conditions to zero, i.e. $\mathbf{X}(0) = \mathbf{0}$ and the uncertainty term ε is negligible because the un-modelled noise accounts for very little extra variations in MMP [28]. Calculating the covariance for both sides of Eq. (3) after the simplification, the variation propagation can be approximated as:

$$\boldsymbol{\Sigma}_{\mathbf{Y}} = \sum_{k=1}^{N} \boldsymbol{\gamma}(k)\boldsymbol{\Sigma}_{\mathbf{U}}(k)\boldsymbol{\gamma}^{T}(k) = \boldsymbol{\Gamma}\boldsymbol{\Sigma}_{\mathbf{U}}\boldsymbol{\Gamma}^{T} \tag{4}$$

where $\boldsymbol{\Sigma}_{\mathbf{Y}}$ and $\boldsymbol{\Sigma}_{\mathbf{U}}(k)$ represent the covariance matrices of $\mathbf{Y}(N)$ and $\mathbf{U}(k)$, $\boldsymbol{\gamma}(k) = \mathbf{C}(N)\boldsymbol{\Phi}(N,k)\mathbf{B}(k)$, $\boldsymbol{\Gamma} = [\boldsymbol{\gamma}(1)\cdots\boldsymbol{\gamma}(N)]$, and $\boldsymbol{\Sigma}_{\mathbf{U}} = diag(\boldsymbol{\Sigma}_{\mathbf{U}}(1), \cdots, \boldsymbol{\Sigma}_{\mathbf{U}}(N))$.

2.3. Distribution of KCC variations considering process degradation

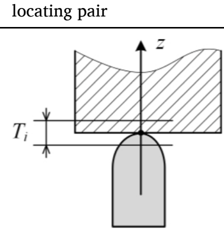
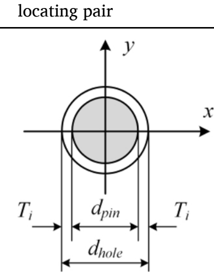
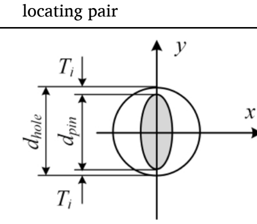
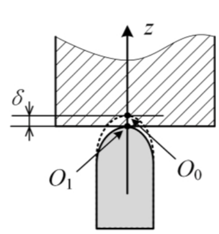
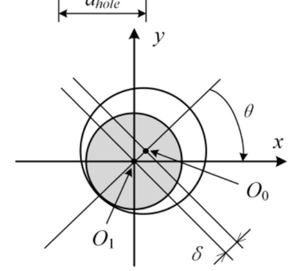
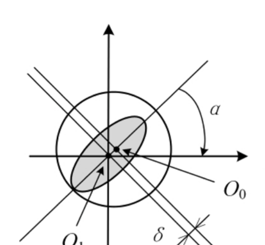
2.3.1. KCC variations in fixture

Locators and pins are two types of KCC in fixture and have differences in contact form, tolerance-variation relation and degradation model. The distribution of KCC variations in two locating schemes will be deduced in this section. The pin in pin-hole locating scheme is a new variation management object in MMPs.

(a) Conventional 3-2-1 locating scheme

The locating pair of conventional 3-2-1 locating scheme is in the form of point-to-surface contact in single direction as shown in Table 1.

Table 1
Different types of locating pair and its variation.

Locating pair	point-to-surface locating pair	four-way pin-hole locating pair	two-way pin-hole locating pair
Diagram			
Deviation			

* O_0 is the ideal position and O_1 is the actual position.

T_i is the tolerance of i^{th} locator, which limits the initial variation range of this KCC. Tighter tolerance means higher manufacturing requirement and cost, so tolerancing tends to allocate tighter tolerances to locators that have a greater impact on KPCs. The variation motion of locator is only in one direction. Here, z direction is taken as the example. The deviation δ_i is between the actual and ideal position of the locator, which is initially bounded by $[-T_i/2, T_i/2]$ and follows a normal distribution of $N\left(0,(T_i/6)^2\right)$. Since $E[\cdot]$ is the expectation operator, the statistics regarding ΔZ before operation are

$$E[\Delta Z] = E[\delta] = 0 \tag{5}$$

$$\sigma_Z^2 = E[\Delta Z^2] = E[\delta^2] = E^2[\delta] + D[\delta] = \frac{T^2}{36} \tag{6}$$

The locator wear causes the process degradation. Refer to maintenance handbooks [29], material wear rate of locator is generally constant during the process. Taking into account the effect of the coating on locator surface, the material wear rate in initial stage is quite low until the coating is deteriorated. Therefore, the locator degradation can be modelled by a quadratic curve w.r.t. operation time for simplicity [27]. The deviation of the locating pair considering degradation can be updated by

$$d_i(\tau) = \delta_i + \Delta_i(\tau) \tag{7}$$

where δ_i is the initial variation determined by the tolerance, $\Delta_i(\tau)$ is the aggregated wear at age τ which is defined by $\Delta_i(\tau) = G_k \cdot \tau^2$ for locator, G_k is a constant indicates the wear rate of locators at stage k. Eq. (7) means that the locating variation changes with the wear of locator.

Therefore, substituting Eq. (7) into (6) can obtain the variance of the locator variation U_i considering process degradation. It can be expressed as

$$\begin{aligned} Var(U_i) &= \sigma_Z^2(\tau) = E[(\delta+\Delta(\tau))^2] = E[\delta^2 + 2\delta\Delta(\tau) + \Delta^2(\tau)] \\ &= E[\delta^2] + 2E[\delta]\mu_{\Delta\tau} + (\mu_{\Delta\tau})^2 + D[\Delta(\tau)] = \frac{T^2}{36} + (\mu_{\Delta\tau})^2 + \tau\cdot\sigma_\Delta^2 \end{aligned} \tag{8}$$

This formula will be iteratively applied to every point-to-surface locating pair in each stage of MMPs, so that fixture part of $\mathbf{\Sigma}_{\text{U}}$ in Eq. (4) can be expressed in terms of tolerance and operation time of corresponding locator.

(b) Pin-hole locating scheme

Besides the point-to-surface locating pair introduced in last part, two types of locating pairs should be considered in pin-hole locating scheme as shown in Table 1: the four-way pin-hole locating pair and the two-way pin-hole locating pair. The contact between pin and datum hole is the form of cylinder-to-cylinder. d_{pin} is the diameter or major axis of the pin and d_{hole} is the diameter of the datum hole. T_i is the tolerance of i^{th} pin, i.e. the upper limit of the clearance δ_i. The clearance is the deviation of hole centre from pin centre, which is initially bounded by $[0, T_i]$ and follows a normal distribution of $N\left(T_i/2,(T_i/6)^2\right)$.

For the four-way pin-hole locating pair, denoting θ as the contact orientation, the deviation in x and y directions are

$$\Delta X = \delta\cos\theta, \Delta Y = \delta\sin\theta \tag{9}$$

The clearance for a four-way locating pair is homogenous in all directions, so the orientation angle θ follows a uniform distribution of $U(0, 2\pi)$. Therefore, the variation statistics associated with a four-way locating pair can be derived as

$$E[\Delta X] = E[\delta\cos\theta] = E[\delta]\cdot E[\cos\theta] = 0 \tag{10}$$

$$E[\Delta Y] = E[\delta\sin\theta] = E[\delta]\cdot E[\sin\theta] = 0 \tag{11}$$

$$\sigma^2_{X,4-\text{way}} = E[\Delta X^2] = E[\delta^2]\cdot E[\cos^2\theta] = (E^2[\delta] + D[\delta])\cdot\frac{1}{2} = \frac{5T^2}{36} \tag{12}$$

$$\sigma^2_{Y,4-\text{way}} = E[\Delta Y^2] = E[\delta^2]\cdot E[\sin^2\theta] = (E^2[\delta] + D[\delta])\cdot\frac{1}{2} = \frac{5T^2}{36} \tag{13}$$

$$\text{Cov}(\Delta X, \Delta Y) = E[\Delta X\Delta Y] = E[\delta^2\sin\theta\cos\theta] = E[\delta^2]\cdot E[\sin\theta]\cdot E[\cos\theta] = 0 \tag{14}$$

For the two-way pin-hole locating pair, the variation is in single direction determined by α, and the deviation in x and y directions are

$$\Delta X = \kappa\cdot\delta\sin\alpha, \Delta Y = \kappa\cdot\delta\cos\alpha \tag{15}$$

where κ is a sign variable, which determines the sign of variation according to which side of the datum hole touches the pin. α is a fixed value according to the orientation of the two-way pin. Therefore, the statistics regarding ΔX and ΔY can be given as

$$E[\Delta X] = E[\kappa\cdot\delta\sin\alpha] = E[\kappa]\cdot E[\delta]\cdot E[\sin\alpha] = 0 \tag{16}$$

$$E[\Delta Y] = E[\kappa\cdot\delta\cos\alpha] = E[\kappa]\cdot E[\delta]\cdot E[\cos\alpha] = 0 \tag{17}$$

$$\sigma^2_{X,2-\text{way}} = E[\kappa^2\cdot\delta^2\sin^2\alpha] = E[\delta^2]\cdot E[\sin^2\alpha] = \frac{5T^2}{18}\sin^2\alpha \tag{18}$$

$$\sigma^2_{Y,2-\text{way}} = E[\kappa^2\cdot\delta^2\cos^2\alpha] = E[\delta^2]\cdot E[\cos^2\alpha] = \frac{5T^2}{18}\cos^2\alpha \tag{19}$$

$$\text{Cov}(\Delta X, \Delta Y) = E[\kappa^2\cdot\delta^2\sin\alpha\cos\alpha] = E[\delta^2]\cdot E[\sin\alpha]\cdot E[\cos\alpha] = \frac{5T^2}{18}\sin\alpha\cos\alpha \tag{20}$$

Since Eq. (7) gives the expression of locating variation under process degradation, a reasonable degradation model for pin-hole locating pair is the focus to obtain $\Delta_i(\tau)$ of Eq. (7). Archard [30] firstly established a sliding wear model to reflect the physical mechanism of tool wear. Based on this research, Jin and Chen [31] proposed a stochastic degradation model. The aggregated wear after age τ is defined as

$$\Delta_i(\tau) = \Delta_i(\tau-1) + \Delta_i^r(\tau) \tag{21}$$

where $\Delta_i^r(\tau)$ is the incremental wear at time τ, and it follows a lognormal distribution of $\text{Lognorm}(\mu_\Delta, \sigma_\Delta^2)$. The mean wear rate μ_Δ for time τ is modelled as

$$\mu_\Delta(\tau) = \mu_0 + \mu_1 e^{-\beta\tau} \tag{22}$$

which consists of two components: μ_0 is the constant wear rate, $\mu_1 e^{-\beta\tau}$ leads to the exponential decrease from initial wear rate $\mu_0 + \mu_1$ to μ_0.

Therefore, the deviation of two types of locating pair are updated. Substituting Eq. (7) into (12), (13), (18) and (19), the variance in two directions can be expressed as

$$\sigma^2_{X,4-\text{way}}(\tau) = E[(\delta+\Delta(\tau))^2\cdot\cos^2\theta] = \frac{1}{2}E[(\delta+\Delta(\tau))^2] \tag{23}$$

$$\sigma^2_{Y,4-\text{way}}(\tau) = E[(\delta+\Delta(\tau))^2\cdot\sin^2\theta] = \frac{1}{2}E[(\delta+\Delta(\tau))^2] \tag{24}$$

$$\sigma^2_{X,2-\text{way}}(\tau) = E[\kappa^2\cdot(\delta+\Delta(\tau))^2\cdot\sin^2\alpha] = \sin^2\alpha\cdot E[(\delta+\Delta(\tau))^2] \tag{25}$$

$$\sigma^2_{Y,2-\text{way}}(\tau) = E[\kappa^2\cdot(\delta+\Delta(\tau))^2\cdot\cos^2\alpha] = \cos^2\alpha\cdot E[(\delta+\Delta(\tau))^2] \tag{26}$$

Since U_i includes deviations in both direction, the variance of the pin variation considering process degradation can be expressed as

$$\text{Var}(U_{i,4-\text{way}}) = \sqrt{(\sigma^2_{X,4-\text{way}}(\tau))^2 + (\sigma^2_{Y,4-\text{way}}(\tau))^2} = \frac{\sqrt{2}}{2}E[(\delta+\Delta(\tau))^2] \tag{27}$$

$$\mathrm{Var}(U_{i,2-\mathrm{way}}) = \sqrt{\left(\sigma^2_{X,2-\mathrm{way}}(\tau)\right)^2 + \left(\sigma^2_{Y,2-\mathrm{way}}(\tau)\right)^2} = E\left[(\delta + \Delta(\tau))^2\right] \tag{28}$$

where $E[(\delta + \Delta(\tau))^2] = E[\delta^2 + 2\delta\Delta(\tau) + \Delta^2(\tau)] = \frac{5T^2}{18} + T\cdot\mu_{\Delta\tau} + (\mu_{\Delta\tau})^2 + \tau\cdot\sigma^2_{\Delta}$.

These two formulas will be iteratively applied to every pin-hole locating pair in each stage of MMPs in pin-hole locating scheme. Together with the variance calculated by Eq. (8) for three locators, the fixture part of $\boldsymbol{\Sigma}_{\mathbf{U}}$ in Eq. (4) can be expressed in terms of tolerance and operation time of corresponding fixture element.

2.3.2. KCC variations of cutting tool

Cutting tool is another important KCC in MMPs. It is generally assumed that the cutting tool is purchased directly without initial variation. During the process, the cutting tool gradually wears and introduces variation. Therefore, the focus of cutting tool variation management is maintenance planning, i.e. cutting tool replacement strategy.

For common machining operations, cutting tool wear is modelled as a third-order polynomial function of operation time in existing researches, while in high-speed machining operations, it tends to follow a second-order polynomial function [32]. Without loss of generality, the cutting tool wear variable in MMPs is assumed to follow a quadratic curve in the form of

$$w_j(\tau) = E_k\cdot\tau + F_k\cdot\tau^2 \tag{29}$$

where $w_j(\tau)$ is the j^{th} cutting tool wear after age τ, E_k and F_k are wear rate coefficients at stage k. Therefore, the cutting tool variation, defined as $w_j(\tau)$, is a random variable with the probability density function derived as follows.

For the j^{th} cutting tool, a^c_j is defined as the decision variable of replacement cycle, thus, $\tau \in [0, a^c_j]$ and $w^{\max}_j$ is the admissible maximum wear of cutting tool when $\tau = a^c_j$. Discretizing the operation time, the variable τ can be considered as a uniform distribution of $U(0, a^c_j)$. Therefore, the probability density function of the j^{th} cutting tool wear can be obtained by Eq. (30). For a function of $y = r(x)$, if r is differentiable and the probability density function of x has already known as $f_X(x)$, the probability density function of y can be calculated as

$$g_Y(y) = f_X(r^{-1}(y))\cdot\left|\frac{dr^{-1}(y)}{dy}\right| \tag{30}$$

Therefore, the probability density function of cutting tool variation is

$$g(w_j) = \frac{1}{a^c_j}\cdot\left|\frac{1}{\sqrt{(E_k)^2 + 4\cdot F_k\cdot w_j}}\right| \tag{31}$$

After enough numerical simulation, the variance of cutting tool variation can be obtained in terms of operation time, which will be iteratively applied to every stage of MMPs so that cutting tool part of $\boldsymbol{\Sigma}_{\mathbf{U}}$ in Eq. (4) can be expressed.

2.4. Optimization model construction

The conventional variation management models are single-objective optimization [20], or the objective function is the sum of quality loss and variation management cost [27]. A bi-objective optimization model solved by the MCGP approach can find the equilibrium point between quality and cost, and it averts the difficulty of specifying the weights of two components [33]. Besides, in addition to dimensional tolerance constraints, geometric tolerance needs to be added for the constraints of the optimization model. The definition of objective functions and constraints, and the solution of the optimization model are introduced in this section.

2.4.1. Objective functions definition

For this bi-objective optimization model, the objective functions include minimizing the variation management cost and minimizing the quality loss.

(a) The objective function of variation management cost

Considering two types of KCC in MMPs, the variation management cost includes:

1) *Fixture cost*: The tolerance-cost function for each locator/pin is reciprocal in this paper, i.e. $C^i_T = w_i/T_i, i = 1, ..., p$, where T_i is the allocated tolerance of i^{th} locator/pin and w_i is the associated weight coefficient. In addition to the tolerance cost, the scheduled maintenance will introduce another fixed cost c^f_{0i} (due to labour and management) for fixture elements. Therefore, the fixture cost includes tolerance cost and fixed cost. The cost induced at time t can be defined as

$$C^f_M(t) = \sum_{i\in S_t}(C^i_T + c^f_{0i}) = \sum_{i\in S_t}\left(\frac{w_i}{T_i} + c^f_{0i}\right), i = 1, ..., p \tag{32}$$

where S_t is the index set of locator/pin subjected to a scheduled maintenance at time t, which is determined by maintenance strategy $\mathbf{a}^f$. $i \in S_t$ means that the i^{th} locator/pin needs to perform the maintenance at time t according to the schedule. Hence, $C^f_M(t)$ depends on both decision parameters and can be described as $C^f_M(t;\mathbf{T},\mathbf{a}^f)$. Tighter tolerance or higher frequency of maintenance will lead to higher cost.

2) *Cutting tool cost*: Similar to the fixture cost, cutting tool cost induced at time t can be defined as

$$C^c_M(t) = \sum_{j\in S_t}(c^c_j + c^c_{0j}), j = 1, ..., q \tag{33}$$

where c^c_j is the cost of cutting tool itself, c^c_{0j} is the fixed cost of j^{th} cutting tool replacement, $j \in S_t$ means that the j^{th} cutting tool needs to replace at time t according to the schedule. Hence, $C^c_M(t)$ depends on the decision parameter of $\mathbf{a}^c$ and can be described as $C^c_M(t;\mathbf{a}^c)$. A high replacement frequency increases the cost.

(b) The objective function of quality loss

The variation propagation model proposed in Section 2.2 can effectively predict the variations of KPCs, denoted as $\mathbf{Y}$. The principle of Taguchi quality loss is "the smaller the better" and the quality loss function is quadratic. Pignatiello [34] extended quality loss function for multi-dimensional variables. In this paper, Eq. (34) is used for multivariate quality loss function:

$$L(\mathbf{Y}) = \mathbf{Y}^T\mathbf{S}\mathbf{Y} \tag{34}$$

where $\mathbf{S} = q\mathbf{I}$ is a symmetric positive-definite matrix defining the contribution of each KPC to product quality loss.

Since $\mathbf{Y}$ depends on the variations of KCCs which is determined by time t and decision parameters $\mathbf{T}$ and $\mathbf{a}$, the quality loss function $L(\mathbf{Y})$ is determined by all three variables, which can be described as $L(\mathbf{Y}(t);\mathbf{T},\mathbf{a})$.

2.4.2. Constraints definition

The constraints of the optimization model are mainly divided into two categories: product quality constraints and decision parameter limit constraints. The former is the focus and will be introduced in detail in this section, while the decision parameter limit constraints will be given directly in the optimization model of next section.

Product tolerances define the design specifications of workpiece. To ensure the product quality, dimensional tolerance and geometric tolerance should be satisfied at the same time during the MMP, that is, both tolerances should be mathematically reflected in constraint equations.

In this paper, it is the first time to expand the scope of variation management of MMP to the geometric scale.

As illustrated in Section 2.2, the KPC deviation vector can be represented by $\mathbf{Y} = \left[(\mathbf{y}_1^R)^T \cdots (\mathbf{y}_m^R)^T\right]^T$, and the deviation of a single KPC can be represented by DMV in the form of $\mathbf{y}_h^R = \left[\Delta x_h^R \Delta y_h^R \Delta z_h^R \Delta \alpha_h^R \Delta \beta_h^R \Delta \gamma_h^R\right]^T$. According to the concept of invariance, tolerances only restrict non-invariant elements of DMV [35]. Planar and cylindrical KPCs account for the vast majority in MMPs. The planar feature has three invariants: two translations parallel to the plane and the rotation around the normal vector of the plane, while the cylindrical feature has two: the translation along and the rotation around its own axis. Hence, the constraint tolerance for planar KPC can be simplified as $[\infty \; \infty w \;\; \alpha \;\; \beta \; \infty]^T$ and that of cylindrical KPC is $[u \;\; v \;\; \infty\alpha \;\; \beta \;\; \infty]^T$, where $\mathbf{T}_h^{KPC} = [u, v, w, \alpha, \beta, \gamma]$ includes six constraint elements in the form of DMV for general features.

According to the variation propagation model of Eq. (4), the variability of a KPC, indexed as h, can be calculated as:

$$\boldsymbol{\Sigma}_{\mathbf{y}_h} = \boldsymbol{\Gamma}_h \boldsymbol{\Sigma}_{\mathbf{U}} \boldsymbol{\Gamma}_h^T, h = 1, \ldots, m \tag{35}$$

where $\boldsymbol{\Sigma}_{\mathbf{y}_h}$ is the 6×6 covariance matrix indicating the deviation of h^{th} KPC, whose six diagonal elements are the variances of six elements in $\mathbf{y}_h = [\Delta x \Delta y \Delta z \Delta \alpha \Delta \beta \Delta \gamma]^T$, i.e., $\boldsymbol{\Sigma}_{\mathbf{y}_h}[\rho, \rho] = \mathrm{Var}(\mathbf{y}_h[\rho]), \rho = 1, 2, \cdots, 6$. To ensure the quality, it should be restricted by the tolerance $\mathbf{T}_h^{KPC}$, which can be denoted by $\boldsymbol{\Sigma}_{\mathbf{y}_h}[\rho, \rho] \le \mathrm{Var}(\mathbf{T}_h^{KPC}[\rho])$.

For dimensional and various geometric tolerances, Loose et al. [12] firstly integrated GD&T and constrained the deviations based on boundary points. However, the number of constraint equations depends on the quantity of boundary points, and each KPC requires at least four boundary points, which means that a large number of nonlinear constraints require a complicated and time-consuming optimization algorithm. Therefore, new constraint method needs to be proposed for $\boldsymbol{\Sigma}_{\mathbf{y}_h}$.

Table 2 shows the general deviations in dimensional and geometric scale for two types of features and builds the constraint relations between DMV elements according to various tolerances. For each non-invariant element, the allowable range is restricted as well. The planar KPC is constrained by parallelism tolerance ε besides dimensional tolerance T_d. Roy and Li [36] proposed the variation constraint inequalities of planar feature. For other geometric tolerances in planar feature, such as angularity and perpendicularity, the solution is to multiply the deviation DMV by a homogeneous transformation matrix. For the cylindrical KPC, the centreline is constrained. The case in Table 2 is marked with the tolerance $\phi\omega$ w.r.t. the GD&T datum features A and B.

The variance of the elements of interest in $\mathbf{T}_h^{KPC}$ is simulated by Monte Carlo method. For instance, w is the focus of planar feature, it has a variance of 0.0126 when T_d =0.1 mm, ε =0.05 mm, a =300 mm, b =100 mm, and the times of simulation is 10000. That means the quality constraint of this KPC is $\boldsymbol{\Sigma}_{\mathbf{Y}_h}[3, 3] \le 0.0126$.

2.4.3. Optimization model

Since the objective functions and constraints are available, this subsection will formulate the optimization model for KCC variation management in MMPs.

The overall variation management cost at time t can be calculated as

$$C(t) = C_M^f(t; \mathbf{T}, \mathbf{a}^f) + C_M^c(t; \mathbf{a}^c) \tag{36}$$

The long-run expected variation management cost per unit time is

$$\begin{aligned}\Phi(\mathbf{T}, \mathbf{a}) &\equiv \lim_{t\to\infty} \frac{\sum_{\tau=0}^{t} E(C(\tau))}{t} = \lim_{t\to\infty} \frac{\sum_{\tau=0}^{t} E\left(\sum_{i\in S_t}\left(\frac{w_i}{T_i} + c_{0i}^f\right) + \sum_{j\in S_t}\left(c_j^c + c_{0j}^c\right)\right)}{t} \\ &= \sum_{i=1}^{p}\sum_{j=1}^{q}\left(\lim_{t\to\infty}\frac{\sum_{\tau=0}^{t}\left(E(C_M^{f,i}(\tau)) + E(C_M^{c,j}(\tau))\right)}{t}\right)\end{aligned} \tag{37}$$

where $C_M^{f,i}(\tau) \equiv \begin{cases} w_i/T_i + c_{0i}^f, & \text{if } i \in S_t \\ 0, & \text{otherwise} \end{cases}$ and $C_M^{c,j}(\tau) \equiv \begin{cases} c_j^c + c_{0j}^c, & \text{if } j \in S_t \\ 0, & \text{otherwise} \end{cases}$.

For the quality loss $L(\mathbf{Y}(t); \mathbf{T}, \mathbf{a})$, the expected value of quality loss is

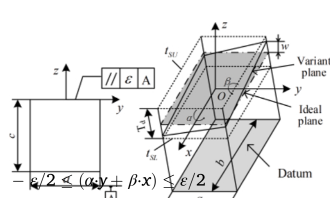
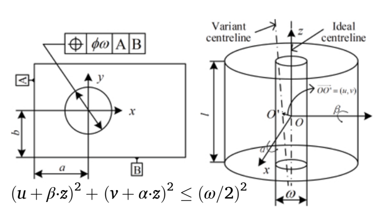
$$\begin{aligned}E(L(\mathbf{Y})) &= E\left(\sum_{i=1}^{m}\sum_{j=1}^{m} s_{ij} Y_i Y_j\right) = \sum_{i=1}^{m}\sum_{j=1}^{m} s_{ij} Cov(Y_i, Y_j) \\ &= \sum_{r=1}^{n}\left(\boldsymbol{\Gamma}_{(:,r)}^T \mathbf{S} \boldsymbol{\Gamma}_{(:,r)}\right)\mathrm{Var}(U_r) = \sum_{r=1}^{n}\rho_r \mathrm{Var}(U_r)\end{aligned} \tag{38}$$

where s_{ij} is the $(i,j)^{\text{th}}$ element of matrix $\mathbf{S}$, $\rho_r \equiv \boldsymbol{\Gamma}_{(:,r)}^T \mathbf{S} \boldsymbol{\Gamma}_{(:,r)}$, and $\rho_r \mathrm{Var}(U_r)$ denotes the contribution of r^{th} KCC to the quality loss.

Therefore, the long-run expected cost per unit time can be obtained by

$$\begin{aligned}\Psi(\mathbf{T}, \mathbf{a}) &\equiv \lim_{t\to\infty}\frac{\sum_{\tau=0}^{t} E(L(\mathbf{Y}(\tau)))}{t} = \lim_{t\to\infty}\frac{\sum_{\tau=0}^{t}\left(\sum_{r=1}^{n}\rho_r \mathrm{Var}(U_r)\right)}{t} \\ &= \sum_{r=1}^{n}\left(\lim_{t\to\infty}\frac{\sum_{\tau=0}^{t} L_r(\mathbf{Y}(\tau))}{t}\right)\end{aligned} \tag{39}$$

Table 2
Constraint relations considering geometric tolerances.

	The planar KPC	The cylindrical KPC
Tolerance condition	dimensional tolerance T_d parallelism tolerance ε	position tolerance $\phi\omega$
Schematic diagram		
Constraint relations	$-\varepsilon/2 \le (\alpha{\cdot}y + \beta{\cdot}x) \le \varepsilon/2$ $-T_d/2 \le (w + \alpha{\cdot}y + \beta{\cdot}x) \le T_d/2$	$(u + \beta{\cdot}z)^2 + (v + \alpha{\cdot}z)^2 \le (\omega/2)^2$
Allowable range of DMV non-invariant elements	$-\varepsilon/b \le \alpha \le \varepsilon/b$ $-\varepsilon/a \le \beta \le \varepsilon/a$ $-T_d/2 \le w \le T_d/2$	$-\omega/2l \le \alpha \le \omega/2l$ $-\omega/2l \le \beta \le \omega/2l$ $-\omega/2 \le u \le \omega/2$ $-\omega/2 \le v \le \omega/2$

The objective of the model is to minimize the variation management cost and minimize quality loss, so that optimal values for all decision parameters of variant management in MMPs can be obtained to achieve quality-cost equilibrium. The optimization model is formulated as

$$\min\begin{cases}\Phi(\mathbf{T},\mathbf{a})=\sum_{i=1}^{p}\sum_{j=1}^{q}\left(\lim_{t\to\infty}\frac{\sum_{\tau=0}^{t}\left(E(C_M^{f,i}(\tau))+E(C_M^{c,j}(\tau))\right)}{t}\right)\\ \Psi(\mathbf{T},\mathbf{a})=\sum_{r=1}^{n}\left(\lim_{t\to\infty}\frac{\sum_{\tau=0}^{t}L_r(\mathbf{Y}(\tau))}{t}\right)\end{cases}\tag{40}$$

subjected to $\boldsymbol{\Sigma}_{\mathbf{y}_h}[\rho,\rho]\leq \mathrm{Var}(\mathbf{T}_h^{KPC}[\rho])$, $0\leq T_i\leq T_i^{\max}$, $a_i^f\geq 0$, $a_j^c\geq 0$, $\forall i,j,r,h,\rho$.

3. Case study

3.1. Problem description

The automotive engine cylinder block machining (see Fig. 4) is a typical MMP that consists of more than twenty stages. The top surface of engine cylinder block serves as the joint surface with the engine cylinder head [37], and its quality seriously affects the sealing performance of the engine [38,39]. Therefore, this paper focuses on the machining of top surface and extracts three representative sequential stages from real production. These three sequential stages can reflect the variation propagation in MMP and contain different kinds of locating pair, which can effectively verify the proposed KCC variation management method.

Fig. 5 briefly describes the three stages. Detail information including the datum features and the nominal locations of machining features w.r.t. reference CS are listed in Table 3. Specifically, OP10 adopts the conventional 3-2-1 locating scheme and T_1, T_2, T_3, U_1, U_2 and W are rough datum features of the workpiece. The deviation of #299 machined at OP10 and the deviations of two holes drilled at other stage are part of variation sources for the machining features at OP20. Similarly, the datum variations from OP20 accumulate deviations at OP30, which construct the SoV. OP20 and OP30 both adopt the pin-hole locating scheme including locating pairs of a four-way pin and a two-way pin. After OP30, the machined workpiece is moved to inspection to measure the KPC defined by the surface #399. The dimensional tolerance for this KPC is ±0.05 mm and a parallelism tolerance is 0.03 mm w.r.t. the GD&T datum features #499.

In this case, 19 KCCs are considered for variation management ($r=1,\cdots,19$), including six locators in OP10 (P_1-P_6), three locators and two pins in both OP20 (P_7-P_{11}) and OP30 ($P_{12}-P_{16}$), and one cutting tool in each stage (Q_1-Q_3). The objective of variation management for MMPs is to assign optimal tolerances and replacement cycles for each KCC to achieve the quality-cost equilibrium.

3.2. Numerical analysis

A numerical analysis was conducted with the parameters presented in Table 4 to optimize the decision parameters **T** and **a**. Several multi-objective optimization methods are available in the literature to solve this model. The weighted method is the most widely used but it involves subjectivity or bias in specifying weights when aggregate incomparable objectives into a single equivalent function [40]. Similarly, the Archimedean and non-Archimedean goal programming methods suffer the difficulty in determining the ranking of the goals in a pre-emptive preference order. The MCGP approach [41] helps to avoid the above difficulties and is adopted in this paper. The underlying philosophy when using the Chebyshev distance metric is that of balance. That is, this method is trying to achieve a good balance between the achievement of the set of goals as opposed to the lexicographic approach which deliberately prioritizes some goals over others or the weighted approach which chooses the set of decision variable values which together make the achievement function lowest [42]. Therefore, the MCGP approach has the potential to give the optimal solution where a balance between the levels of satisfaction of the goals is needed [43].

For specific steps, first decompose the model into two single-objective optimization problems under the same constraints. Each one is solved by *fmincon* function in MATLAB which adopts the method of sequential quadratic programming (SQP). The SQP method is an efficient nonlinear programming algorithm and has the property of fast convergence. The solution of SQP is equivalent to solving a sequence of quadratic sub-problems. The objective values of the solutions to these two decomposed problems can be expressed as Table 5.

The values of two objectives cannot be compared directly. Minimizing variation management cost means a higher quality loss, and reducing quality loss leads to an increase of cost. Since neither situation is desirable, it calls for the MCGP method to obtain a compromised solution. It can be interpreted by fuzzy programming terms, and the fuzzy membership functions of two objectives are expressed as [44]

$$\mu_1=\begin{cases}1,\ \Phi(\mathbf{T},\mathbf{a})\leq\Phi(\mathbf{T},\mathbf{a})_1^*\\ \dfrac{\Phi(\mathbf{T},\mathbf{a})_2^*-\Phi(\mathbf{T},\mathbf{a})}{\Phi(\mathbf{T},\mathbf{a})_2^*-\Phi(\mathbf{T},\mathbf{a})_1^*},\ \Phi(\mathbf{T},\mathbf{a})_1^*\leq\Phi(\mathbf{T},\mathbf{a})\leq\Phi(\mathbf{T},\mathbf{a})_2^*\\ 0,\ \Phi(\mathbf{T},\mathbf{a})\geq\Phi(\mathbf{T},\mathbf{a})_2^*\end{cases}\tag{41}$$

$$\mu_2=\begin{cases}1,\ \Psi(\mathbf{T},\mathbf{a})\leq\Psi(\mathbf{T},\mathbf{a})_2^*\\ \dfrac{\Psi(\mathbf{T},\mathbf{a})-\Psi(\mathbf{T},\mathbf{a})_1^*}{\Psi(\mathbf{T},\mathbf{a})_2^*-\Psi(\mathbf{T},\mathbf{a})_1^*},\ \Psi(\mathbf{T},\mathbf{a})_2^*\leq\Psi(\mathbf{T},\mathbf{a})\leq\Psi(\mathbf{T},\mathbf{a})_1^*\\ 0,\ \Psi(\mathbf{T},\mathbf{a})\geq\Psi(\mathbf{T},\mathbf{a})_1^*\end{cases}\tag{42}$$

The best distance from two objectives' worst values is the

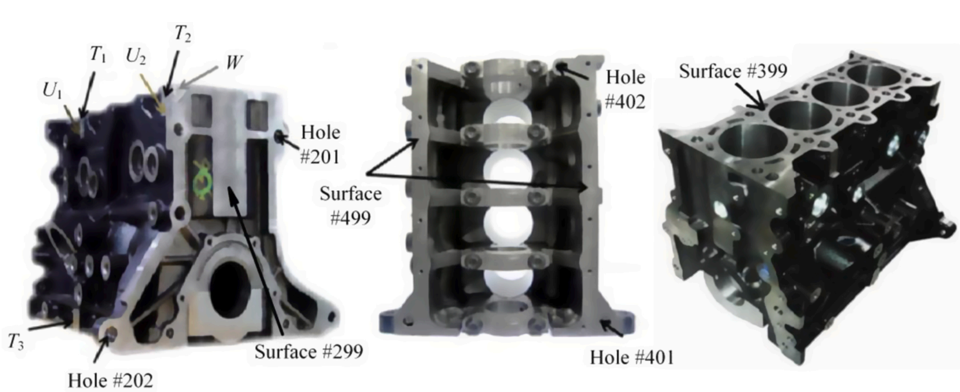


Fig. 4. The automotive engine cylinder block.

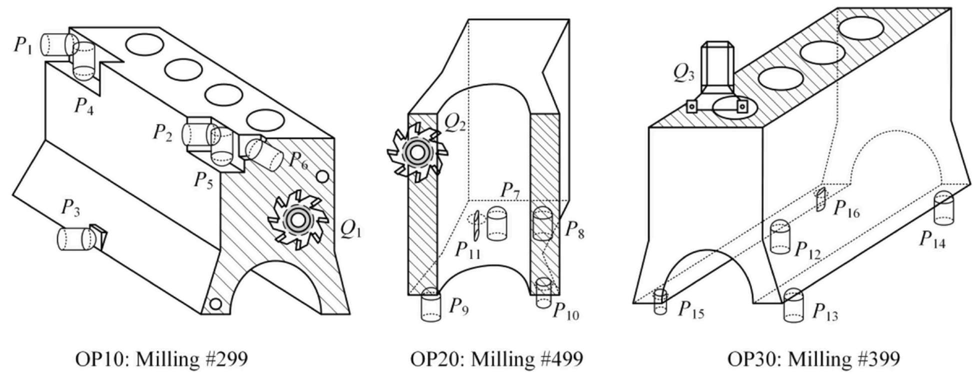


Fig. 5. Three stages of the MMP.

Table 3
Process description and nominal locations of key features.

Stage	Datum features	Process descriptions	${}^{0}R_{0}$	${}^{0}\mathbf{t}_{0}^{R}$
OP10	T_1, T_2, T_3, U_1, U_2, W	Mill flank surface #299	[0,0,0]	[-170.5,138,0]
OP20	#299, #201, #202	Mill bottom surface #499	[0,π/2,π/2]	[17.5,248,-30.5]
OP30	#499, #401, #402	Mill top surface #399	[0,-π/2,π/2]	[-218.5,248,-30.5]

compromised solution which can be obtained by maximizing δ. It is formulated as

$$\max \delta \tag{43}$$

subjected to $\delta \leq \frac{\Phi(\mathbf{T},\mathbf{a})_2^* - \Phi(\mathbf{T},\mathbf{a})}{\Phi(\mathbf{T},\mathbf{a})_2^* - \Phi(\mathbf{T},\mathbf{a})_1^*}$, $\delta \leq \frac{\Psi(\mathbf{T},\mathbf{a}) - \Psi(\mathbf{T},\mathbf{a})_1^*}{\Psi(\mathbf{T},\mathbf{a})_2^* - \Psi(\mathbf{T},\mathbf{a})_1^*}$, and all constraints in original model.

The solution to the model is the equilibrium point of quality and cost, specifically, $\delta = 0.814$ with $\Phi(\mathbf{T},\mathbf{a})^* = 1.820$ and $\Psi(\mathbf{T},\mathbf{a})^* = 0.279$. The optimal tolerance and replacement cycle for each KCC are listed in Table 6. The variation management strategies for diverse KCCs are different because each KCC contributes differently to the quality loss. For example, the variation of locator P_4 has little impact on the quality of final workpiece, a looser tolerance and a longer replacement cycle are assigned, which can help to reduce cost while the quality is still kept at a desirable level.

3.3. Comparison and discussion

Corresponding to the three major contributions of this paper, the superiority of the proposed method is compared and discussed in three aspects: (i) application scenarios, (ii) geometric tolerance integration, and (iii) manufacturing system performance.

For the application scenarios comparison, two types of locating schemes are involved in this case. However, the existing KCC variation management researches for MMPs can only handle the conventional 3-2-1 locating scheme in the form of point-to-surface contact. This paper derives the variation distribution for the new KCC type in degradation state and adds it to the new variation management model, which overcomes this limitation and expands the applicable scenarios.

In order to demonstrate the improvement of the variation management method after integrating geometric tolerance, 10000 Monte Carlo simulations are run for two MMPs. The two MMPs for this case are defined by the output of variation management strategy, one of which, as in previous studies, does not consider geometric tolerances, and the other adopts the strategy obtained by the proposed method. A random value for each KCC is generated according to its probability density function for each simulation. To simulate the effects of un-modelled noise, a random variable that follows the normal distribution of $N\left(0, (0.005/6)^2\right)$ is added to each simulation. The simulation results show that the proposed method keeps the 99.21 % of the workpieces conforming to the final quality specifications while only 90.44 % can be ensured if geometric tolerance is not integrated in the variation management strategy. There is a certain difference in the proportion of qualified workpieces because the lack of geometric tolerance consideration leads to looser tolerances while establishing optimization problems. Therefore, by considering the effects of geometric tolerance on MMPs, the variation management method improves the number of workpieces within specifications by 9.7 %, indicating the necessity of geometric tolerance integration. Besides, Fig. 6 shows the probability density distribution of KPC deviation after the Monte Carlo simulations. It confirms that, for the case study, the KPC variability can be approximated as a normal distribution despite that some KCCs are not normally distributed.

For the further comparison of manufacturing system performance, two common approaches for KCC variation management in today's

Table 5
Solution of two single-objective problems.

	min $\Phi(\mathbf{T},\mathbf{a})$	min $\Psi(\mathbf{T},\mathbf{a})$	|Difference|
Value of $\Phi(\mathbf{T},\mathbf{a})$	$\Phi(\mathbf{T},\mathbf{a})_1^* = 1.678$	$\Phi(\mathbf{T},\mathbf{a})_2^* = 2.441$	0.763
Value of $\Psi(\mathbf{T},\mathbf{a})$	$\Psi(\mathbf{T},\mathbf{a})_1^* = 0.297$	$\Psi(\mathbf{T},\mathbf{a})_2^* = 0.275$	0.022

Table 4
Parameters for variation management in the case study.

point-to-surface locating pair related (mm)		pin-hole locating pair related (mm)		cutting tool related (mm)		cost function related ($)		quality loss related (unit/mm^2)		constrain related (mm)	
G_k	1.9×10^{-7}	μ_0	5×10^{-7}	E_k	0.0113	w_i	200	q	1	a	108
μ_Δ	8.7×10^{-8}	μ_1	1×10^{-6}	F_k	0.0019	c_{0i}^{f}	200			b	326.5
σ_Δ	2.0×10^{-7}	β	1×10^{-3}	μ_Δ	3.4×10^{-4}	c_j^{c}	50			$T_i^{\max}$	0.1
		μ_Δ	5.0×10^{-7}	σ_Δ	6.8×10^{-3}	c_{0j}^{c}	100				
		σ_Δ	5.0×10^{-5}								

Table 6
Optimal variation management strategy for each KCC.

i	1	2	3	4	5	6	7	8	9	10
T_i	0.028	0.021	0.031	0.093	0.087	0.016	0.031	0.028	0.029	0.018
α_i^f	3.43	2.32	3.67	11.82	10.18	1.96	3.41	3.58	3.21	1.98
i	11	12	13	14	15	16	j	1	2	3
T_i	0.013	0.029	0.025	0.027	0.092	0.088	α_j^c	4.10	4.94	5.11
α_i^f	1.51	3.54	3.11	3.22	11.27	11.0				

*The unit of T is "mm", the unit of α_i^f is "$\times 10^4$ operations", and the unit of α_j^c is "$\times 10^3$ operations".

automotive industry are considered. The first is purely based on experience and uniformly sets the tolerances of fixture elements to be 0.1 mm. The second is the process-oriented tolerancing method, which provides specific tolerance for each fixture element to deliver a high-quality product. The replacement cycle of fixture element is fixed to 10000 operations and that of cutting tool is fixed to 1000 operations based on experience. The proposed method integrates tolerance and maintenance planning for KCC and the comparison with above two conventional methods are shown in Table 7.

Compared with the engineering experience approach, the process-oriented tolerancing method optimizes the tolerance configuration based on the impact of each KCC on product quality, so that the long-term average quality loss is much smaller without the increase of total cost. This reflects the significance of tolerance allocation for KCC in manufacturing system. For the method proposed in this paper, the maintenance planning is added to the process-oriented tolerancing to form a complete concept of variation management. Although this method suffers a much higher cost at the first setup due to its tighter tolerance, it has a similar long-term average KCC cost to the process-oriented tolerancing method because of a longer service time. More

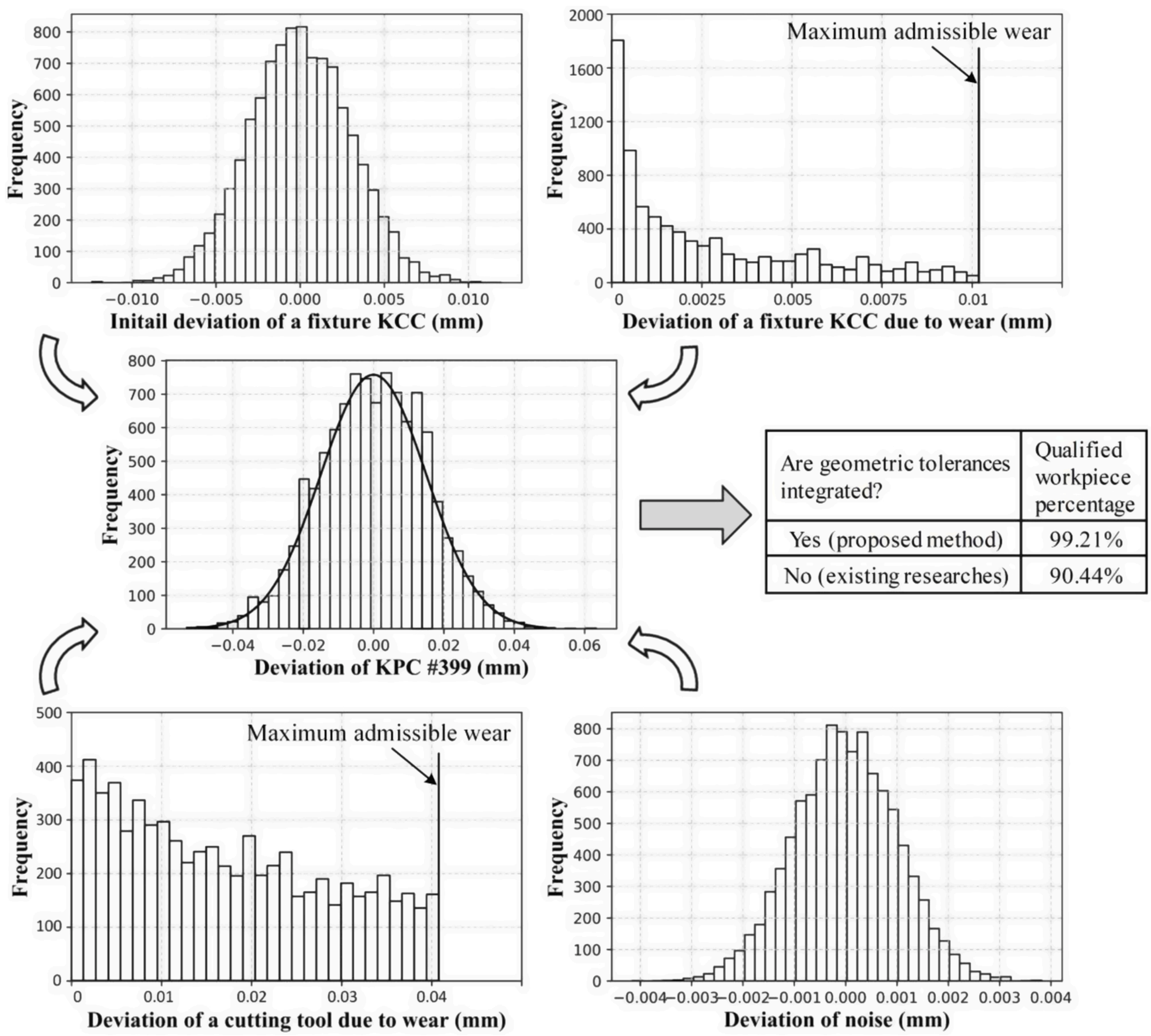


Fig. 6. The distribution of KCCs and KPC deviation after 10000 simulations.

Table 7
Comparison with conventional methods in manufacturing industry.

Item	Formula	Engineering experience method	Process-oriented tolerancing method	The proposed method
KCC cost at the first setup ($)	$\sum_i^p \frac{w_i}{T_i} + \sum_j^q c_j^c$	32000	30628	113955
long-term average KCC cost ($/operation)	$\lim_{t\to\infty} \frac{\sum_{\tau=0}^{t} E\left(\sum_{i\in S_\tau} \frac{w_i}{T_i} + \sum_{j\in S_\tau} c_j^c\right)}{t}$	1.380	1.321	1.394
long-term average variation management cost ($/operation)	$\lim_{t\to\infty} \frac{\sum_{\tau=0}^{t} E\left(\sum_{i\in S_\tau} (\frac{w_i}{T_i} + c_{0i}^f) + \sum_{j\in S_\tau} (c_j^c + c_{0j}^c)\right)}{t}$	2.783	2.724	1.820
long-term average quality loss (unit/operation)	$\lim_{t\to\infty} \frac{\sum_{\tau=0}^{t} E(L(\mathbf{Y}(\tau)))}{t}$	0.554	0.281	0.279

*The KCC cost includes the tolerance cost of fixture element and the cost of cutting tool itself.

important, the proposed method has a significant advantage in the long-term average maintenance cost due to the lower maintenance frequency, which is ultimately reflected in a 33.2 % saving in total variation management cost. For the quality, process-oriented tolerancing significantly improves product quality and reduces the long-term average quality loss by 49.3 %. The proposed method maintains a similar quality loss level while greatly reducing the cost of variation management. Therefore, in terms of quality and cost, the method proposed in this paper achieves the equilibrium and performs better than the two conventional methods wildly adopted in manufacturing industry.

From the deviation measurement results of KPCs obtained in actual production, the adoption of proposed method has also improved the performance of the manufacturing system. According to the calculation of process capacity from real production workshop, the process capacity index of the top surface machining is increased to more than 1.33, and the defective rate is reduced from 0.27 % to 0.11 %. The annual output of this type of engine cylinder block is 350,000 units and the cost of a single one is nearly 70 USD. Simply by reducing defective rate, the annual accumulated cost can be saved by nearly 400,000 USD, not to mention the cost reduction brought by the optimization of maintenance strategy. Therefore, the superiority of the proposed method has been fully discussed in this section, which can greatly improve the manufacturing system performance in terms of both quality and cost.

4. Conclusion

This paper presents a new variation management framework for KCCs in MMPs considering quality-cost equilibrium. The new concept of variation management consists of process-oriented tolerancing and maintenance planning. In the state space model based MMPs, the variation distribution for each KCC is derived in its degradation state. The optimization model assigns the optimal variation management strategy for each KCC based on its impact to the manufacturing system. The limitations in existing researches, such as application scenario, quality specification integrity, and quality-cost equilibrium are fully elaborated and resolved. A case study of automotive engine cylinder block MMP is conducted to demonstrate the potentials of the proposed variation management methodology. The results reveal that, the missing of geometric tolerance constraint when assigning the variation management strategy will lead to a decrease in workpiece qualification rate, and the proposed method can achieve desirable manufacturing system performance in terms of quality and cost with multiple locating schemes covered.

In this paper, a three-stage automotive engine cylinder block machining process is adopted as the example to demonstrate the proposed method. It is worth noting that the overall framework, the state space model for variation propagation, and the KCC degradation model are all quite general for various types of MMPs, which means it has strong scalability for handling various production scenarios. As future work, the machining parameters can be included in the research. Optimizing the machining parameters can effectively improve production efficiency and reduce production costs. However, the adjustment of the machining parameters may bring about problems such as the increase in KCC degradation rate, which will also cause a decline in product quality. Hence, the simultaneous optimization with machining parameters is an interesting issue for variation management. Secondly, preventive maintenance can update the maintenance schedule based on the online measurement of process conditions, reflecting the dynamics of the maintenance. Therefore, the choice of maintenance methodology based on the consideration of measurement cost and feasibility is also a subject worthy of further study.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] Shi J. Stream of variation modeling and analysis for multistage manufacturing processes. New York: CRC press; 2006.
[2] Ding Y, Jin J, Ceglarek D, Shi J. Process-oriented tolerancing for multi-station assembly systems. IIE Trans 2005;37(6):493–508.
[3] Zhou S, Huang Q, Shi J. State space modeling of dimensional variation propagation in multistage machining process using differential motion vectors. IEEE Trans Robot Autom 2003;19(2):296–309.
[4] Loose J-P, Zhou S, Ceglarek D. Kinematic analysis of dimensional variation propagation for multistage machining processes with general fixture layouts. IEEE Trans Autom Sci Eng 2007;4(2):141–52.
[5] Abellán-Nebot JV, Liu J. Variation propagation modelling for multi-station machining processes with fixtures based on locating surfaces. Int J Prod Res 2013;51(15):4667–81.
[6] Yang F, Jin S, Li Z. A modification of DMVs based state space model of variation propagation for multistage machining processes. Assem Autom 2017;37(4):381–90.
[7] Abellán-Nebot JV, Liu J, Subirón FR, Shi J. State space modeling of variation propagation in multistation machining processes considering machining-induced variations. J Manuf Sci Eng Trans ASME 2012;134(2):021002.
[8] Wang K, Du S, Xi L. Three-dimensional tolerance analysis modelling of variation propagation in multi-stage machining processes for general shape workpieces. Int J Precis Eng Manuf 2020;21(1):31–44.
[9] Du S, Yao X, Huang D. Engineering model-based Bayesian monitoring of ramp-up phase of multistage manufacturing process. Int J Prod Res 2015;53(15):4594–613.
[10] Du S, Yao X, Huang D, Wang M. Three-dimensional variation propagation modeling for multistage turning process of rotary workpieces. Comput Ind Eng 2015;82:41–53.
[11] Wang K, Li G, Du S, Xi L, Xia T. State space modelling of variation propagation in multistage machining processes for variable stiffness structure workpieces. Int J Prod Res 2020:1–20.
[12] Loose J-P, Zhou Q, Zhou S, Ceglarek D. Integrating GD&T into dimensional variation models for multistage machining processes. Int J Prod Res 2010;48(11):3129–49.

[13] Chen Y, Ding Y, Jin J, Ceglarek D. Integration of process-oriented tolerancing and maintenance planning in design of multistation manufacturing processes. IEEE Trans Autom Sci Eng 2006;3(4):440–53.
[14] Cai W. A new tolerance modeling and analysis methodology through a two-step linearization with applications in automotive body assembly. J Manuf Syst 2008;27(1):26–35.
[15] Andolfatto L, Thiebaut F, Lartigue C, Douilly M. Quality- and cost-driven assembly technique selection and geometrical tolerance allocation for mechanical structure assembly. J Manuf Syst 2014;33(1):103–15.
[16] Tsutsumi D, Gyulai D, Kovacs A, Tipary B, Ueno Y, Nonaka Y, et al. Joint optimization of product tolerance design, process plan, and production plan in high-precision multi-product assembly. J Manuf Syst 2020;54:336–47.
[17] Jin S, Zheng C, Yu K, Lai X. Tolerance design optimization on cost–quality trade-off using the Shapley value method. J Manuf Syst 2010;29(4):142–50.
[18] Zheng C, Jin S, Lai X, Yu K. Assembly tolerance allocation using a coalitional game method. Eng Optim 2011;43(7):763–78.
[19] Awad MI, Hassan NM. Joint decisions of machining process parameters setting and lot-size determination with environmental and quality cost consideration. J Manuf Syst 2018;46:79–92.
[20] Huang Q, Shi J. Simultaneous tolerance synthesis through variation propagation modeling of multistage manufacturing processes. 31st North American Manufacturing Research Conference 2003:515–22.
[21] Liu J, Shi J, Hu SJ. Quality-assured setup planning based on the stream-of-variation model for multi-stage machining processes. IIE Trans 2009;41(4):323–34.
[22] Chen S, Wang H, Huang Q. Multistage machining process design and optimization using error equivalence method. ASME International Manufacturing Science & Engineering Conference 2009.
[23] Zhang X, Wang H, Chen S, Huang Q. A novel two-stage optimization approach to machining process selection using error equivalence method. J Manuf Syst 2018;49:36–45.
[24] Celen M, Djurdjanovic D. Integrated maintenance and operations decision making with imperfect degradation state observations. J Manuf Syst 2020;55:302–16.
[25] Alimian M, Ghezavati V, Tavakkoli-Moghaddam R. New integration of preventive maintenance and production planning with cell formation and group scheduling for dynamic cellular manufacturing systems. J Manuf Syst 2020;56:341–58.
[26] Hu J, Jiang Z, Liao H. Joint optimization of job scheduling and maintenance planning for a two-machine flow shop considering job-dependent operating condition. J Manuf Syst 2020;57:231–41.
[27] Abellán-Nebot JV, Liu J, Subirón FR. Process-oriented tolerancing using the extended stream of variation model. Comput Ind 2013;64(5):485–98.
[28] Ding Y, Ceglarek D, Shi J. Modeling and diagnosis of multistage manufacturing processes: part I state space model. Japan-USA Symposium of Flexible Automation 2000.
[29] Bloch HP, Geitner FK. Practical machinery management for process plants. Vol 3: machinery component maintenance and repair. Burlington, MA, USA/Oxford, UK: Gulf Professional Publishing; 1985.
[30] Archard JF. Contact and rubbing of flat surfaces. J Appl Phys 1953;24(8):981–8.
[31] Jin J, Chen Y. Quality and reliability information integration for design evaluation of fixture system reliability. Qual Reliab Eng Int 2001;17(5):355–72.
[32] ASM-Handbook. ASM metals handbook. Vol. 16: machining. ASM International; 2003.
[33] Wang P, Liang M. Simultaneously solving process selection, machining parameter optimization, and tolerance design problems: a bi-criterion approach. J Manuf Sci Eng Trans ASME 2005;127(3):533–44.
[34] Pignatiello JJ. Strategies for robust multiresponse quality engineering. IIE Trans 1993;25(3):5–15.
[35] Prisco U, Giorleo G. Overview of current CAT systems. Integr Comput Aided Eng 2002;9(4):373–87.
[36] Roy U, Li B. Representation and interpretation of geometric tolerances for polyhedral objects. II.: size, orientation and position tolerances. Comput Aided Des 1999;31(4):273–85.
[37] Shao Y, Wang K, Du S, Xi L. High definition metrology enabled three dimensional discontinuous surface filtering by extended tetrolet transform. J Manuf Syst 2018;49:75–92.
[38] Shao Y, Yin Y, Du S, Xia T, Xi L. Leakage monitoring in static sealing interface based on three dimensional surface topography indicator. J Manuf Sci Eng 2018;140(10):101003.
[39] Shao Y, Yin Y, Du S, Xi L. A surface connectivity-based approach for leakage channel prediction in static sealing interface. J Tribol 2019;141(6):062201.
[40] Lin Y, Chang P, Yeng L, Huan S. Bi-objective optimization for a multistate job-shop production network using NSGA-II and TOPSIS. J Manuf Syst 2019;52:43–54.
[41] Zimmermann HJ. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets Syst 1978;1(1):45–55.
[42] Jones D, Tamiz M. Practical goal programming. New York: Springer; 2010.
[43] Wang P, Liang M. An integrated approach to tolerance synthesis, process selection and machining parameter optimization problems. Int J Prod Res 2005;43(11):2237–62.
[44] Sakawa M, Kubota R. Fuzzy programming for multiobjective job shop scheduling with fuzzy processing time and fuzzy duedate through genetic algorithms. Eur J Oper Res 2000;120(2):393–407.